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A fast optimization method with the speed of light

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Abstract By using purely resistive circuit, this paper proposes a fast optimization method that can obtain the optimal solution with the speed of light.

Keywords Fast optimization method · Resistive circuit · The speed of light

1 Introduction

Let's consider the following optimization problem

$$\min_{\boldsymbol{x}\in R^n} f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A\boldsymbol{x} - \boldsymbol{x}^T\boldsymbol{b} \tag{1}$$

where $A \in R^{n\times n}$ is a symmetric positive definite matrix, $\boldsymbol{b} \in R^n$ is a vector.

It is known that the optimization problem is equivalent to solving the following linear equations

$$A\boldsymbol{x} = \boldsymbol{b} \tag{2}$$

The linear equations is usually solved by the conjugate gradient method, which is a numerical iteration method and has the time complexity of $\mathscr{O}(m\sqrt{K})$ [1], where m is the number of non-zero entries in A and K is the condition number of A. However, the running time of the conjugate gradient method for the linear equations is not only dependent on the time complexity but also dependent on the space complexity in practice. For example, the convergence rate of Newton's method is quadratic, but it does not mean that the running time of Newton's method is faster than that of first order methods since the calculation of the Hessian matrix is more expensive.

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2 Proposed method

To accelerate the speed of optimization process, in this study, a fast optimiation method based on purely resistive circuit is proposed, and the solution to the optimization optimization problem can be obtained instantly when the power is turned on. Since the speed of the transmission of the electrical current is about the speed of light, the proposed optimization method has the speed of light [2]. In the sequel, several purely resistive circuits are designed to achive the goals.

2.1 Two-dimensional problem

Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

where $a_{12} = a_{21}$.

A purely resistive circuit is designed for the two-dimensional problem, as shown in Fig. 1.

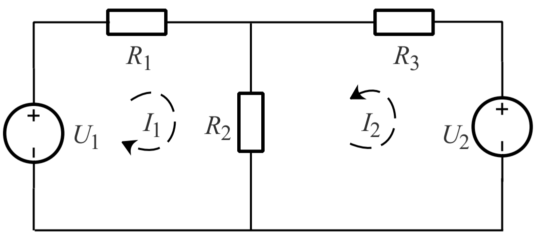

Fig. 1 Purely resistive circuit for the two-dimensional problem

By using Kirchhoff's circuit laws and the mesh current method, we have

$$\begin{aligned} R_1 I_1 + R_2(I_1 + I_2) &= U_1 \\ R_2(I_1 + I_2) + R_3 I_2 &= U_2 \end{aligned} \tag{3}$$

which is equivalent to

$$\begin{aligned} (R_1 + R_2)I_1 + R_2 I_2 &= U_1 \\ R_2 I_1 + (R_2 + R_3)I_2 &= U_2 \end{aligned} \tag{4}$$

then, we have

$$R_1 = a_{11} - a_{12}, \quad R_2 = a_{12}, \quad R_3 = a_{22} - a_{12}, \quad U_1 = b_1, \quad U_2 = b_2 \tag{5}$$

that is to say, by desiging the purely resistive circuit as shown in Fig. 1, the optimal solution to the optimization problem can be instantly obtained by $(x_1^*, x_2^*) = (I_1, I_2)$, where, I_1 and I_2 are the currents through resistors R_1 and R_3, respetively.

Example 1 Considering the following two-dimensional instance

$$\min_{\boldsymbol{x} \in R^n} f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A \boldsymbol{x} - \boldsymbol{x}^T \boldsymbol{b}$$

where,

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}.$$

Then, we have $R_1 = 2, R_2 = 1, R_3 = 3$ and $U_1 = 5, U_2 = 9$. The circuit simulation using Multisim for this problem is given in Fig. 2, and it is found that the solution to the optimization problem can be obtained instantly as $x^* = (1.000, 2.000)$

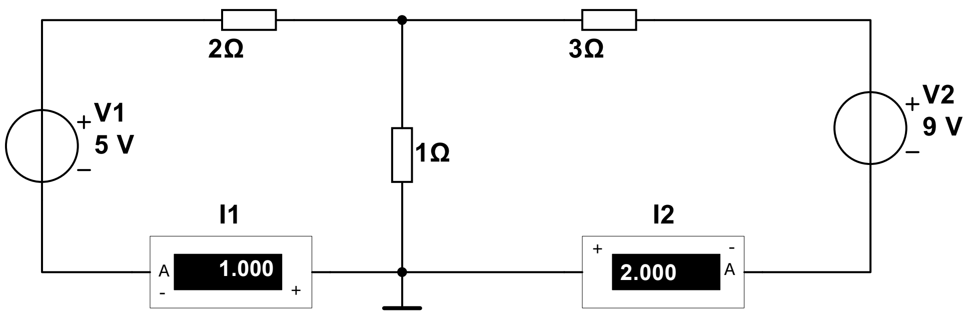

Fig. 2 The circuit simulation using Multisim for the two-dimensional instance

2.2 Three and four dimensional problems

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$

where $a_{12} = a_{21}, a_{13} = a_{31}, a_{23} = a_{32}$.

A purely resistive circuit is designed for the three-dimensional problem, as shown in Fig. 3.

By using Kirchhoff's circuit laws and the mesh current method, we have

$$\begin{aligned} (R_1 + R_2 + R_4)I_1 + R_2 I_2 + R_4 I_3 &= U_1 \\ R_2 I_1 + (R_2 + R_3 + R_5)I_2 - R_5 I_3 &= U_2 \\ R_4 I_1 - R_5 I_2 + (R_4 + R_5 + R_6)I_3 &= U_3 \end{aligned} \tag{6}$$

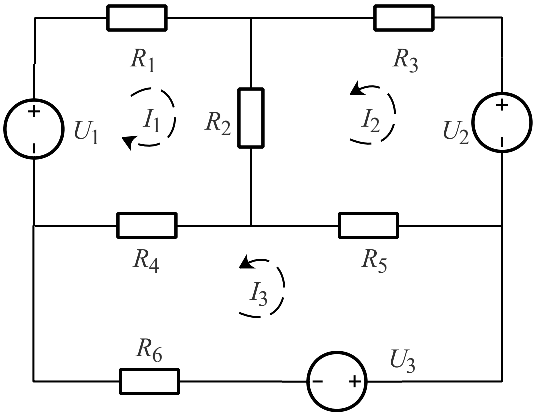

Fig. 3 Purely resistive circuit for the three-dimensional problem

then, we have

$$\begin{aligned} &R_1 = a_{11} - a_{12} - a_{13},\ R_2 = a_{12},\ R_3 = a_{22} + a_{23} - a_{12} \\ &R_4 = a_{13},\ R_5 = -a_{23},\ R_6 = a_{33} + a_{23} - a_{13} \\ &U_1 = b_1,\ U_2 = b_2,\ U_3 = b_3 \end{aligned} \tag{7}$$

that is to say, by desiging the purely resistive circuit as shown in Fig. 3, the optimal solution to the optimization problem can be instantly obtained by $(x_1^*, x_2^*, x_3^*) = (I_1, I_2, I_3)$, where, I_1 and I_2 are the currents through resistors R_1, R_3 and R_6, respetively.

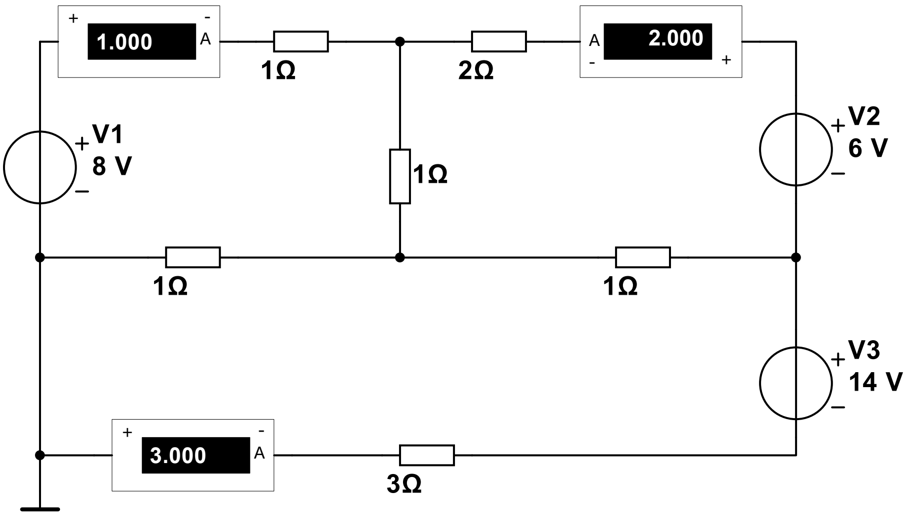

Fig. 4 The circuit simulation using Multisim for the three-dimensional instance

Example 2 Considering the following three-dimensional instance

$$\min_{\boldsymbol{x}\in R^n} f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A\boldsymbol{x} - \boldsymbol{x}^T\boldsymbol{b}$$

where,

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 4 & -1 \\ 1 & -1 & 5 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 8 \\ 6 \\ 14 \end{bmatrix}.$$

Then, we have $R_1 = 1, R_2 = 1, R_3 = 2, R_4 = 1, R_5 = 1, R_6 = 3$, and $U_1 = 8, U_2 = 6, U_3 = 14$. The circuit simulation using Multisim for this problem is given in Fig. 4, and it is found that the solution to the optimization problem can be obtained instantly as $x^* = (1.000, 2.000, 3.000)$.

Similarly, let Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix},$$

where $a_{12} = a_{21}, a_{13} = a_{31}, a_{14} = a_{41}, a_{23} = a_{32}, a_{24} = a_{42}, a_{34} = a_{43}$.

A purely resistive circuit is designed for the four-dimensional problem, as shown in Fig. 5.

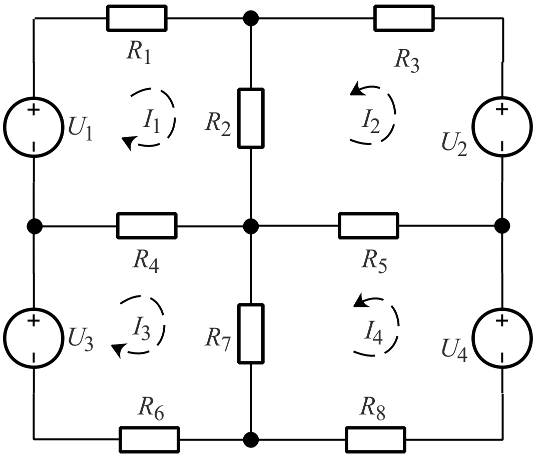

Fig. 5 Purely resistive circuit for the four-dimensional problem

By using Kirchhoff's circuit laws and the mesh current method, we have

$$\begin{aligned} (R_1+R_2+R_4)I_1 + R_2 I_2 - R_4 I_3 &= U_1 \\ R_2 I_1 + (R_2+R_3+R_5)I_2 - R_5 I_4 &= U_2 \\ -R_4 I_1 + (R_4+R_6+R_7)I_3 + R_7 I_4 &= U_3 \\ -R_5 I_2 + R_7 I_3 + (R_5+R_7+R_8)I_4 &= U_4 \end{aligned} \tag{8}$$

then, we have

$$
\begin{aligned}
&R_1 = a_{11} + a_{13} - a_{12},\ \ R_2 = a_{12},\ \ R_3 = a_{22} + a_{24} - a_{12} \\
&R_4 = -a_{13},\ \ R_5 = -a_{24},\ \ R_6 = a_{13} + a_{33} - a_{34} \\
&R_7 = a_{34},\ \ R_8 = a_{24} + a_{44} - a_{34}, \\
&U_1 = b_1,\ \ U_2 = b_2,\ \ U_3 = b_3,\ \ U_4 = b_4 \\
&a_{14} = a_{41} = a_{23} = a_{32} = 0
\end{aligned} \tag{9}
$$

that is to say, by desiging the purely resistive circuit as shown in Fig. 5, the optimal solution to the optimization problem can be instantly obtained by $(x_1^*, x_2^*, x_3^*, x_4^*) = (I_1, I_2, I_3, I_4)$, where, I_1, I_2, I_3 and I_4 are the currents through resistors R_1, R_3, R_6 and R_8, respetively.

Example 3 Considering the following four-dimensional instance

$$\min_{\boldsymbol{x} \in R^n} f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A \boldsymbol{x} - \boldsymbol{x}^T \boldsymbol{b}$$

where,

$$A = \begin{bmatrix} 5 & 1 & -1 & 0 \\ 1 & 6 & 0 & -1 \\ -1 & 0 & 7 & 1 \\ 0 & -1 & 1 & 8 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 4 \\ 9 \\ 24 \\ 33 \end{bmatrix}.$$

Then, we have $R_1 = 3, R_2 = 1, R_3 = 4, R_4 = 1, R_5 = 1, R_6 = 5, R_7 = 1, R_8 = 6$. $U_1 = 4, U_2 = 9, U_3 = 24, U_4 = 33$. The circuit simulation using Multisim for this problem is given in Fig. 6, and it is found that the solution to the optimization problem can be obtained instantly as $x^* = (1.000, 2.000, 3.000, 4.000)$.

3 Conclusion and future work

In this study, a fast optimization method was proposed based on the purely resistive circuit, and this optimization method has the speed of light in theory. Several instances are given to show the feasiblity the proposed method. In the future, other circuit topologies will be investigated for large scale optimization.

References

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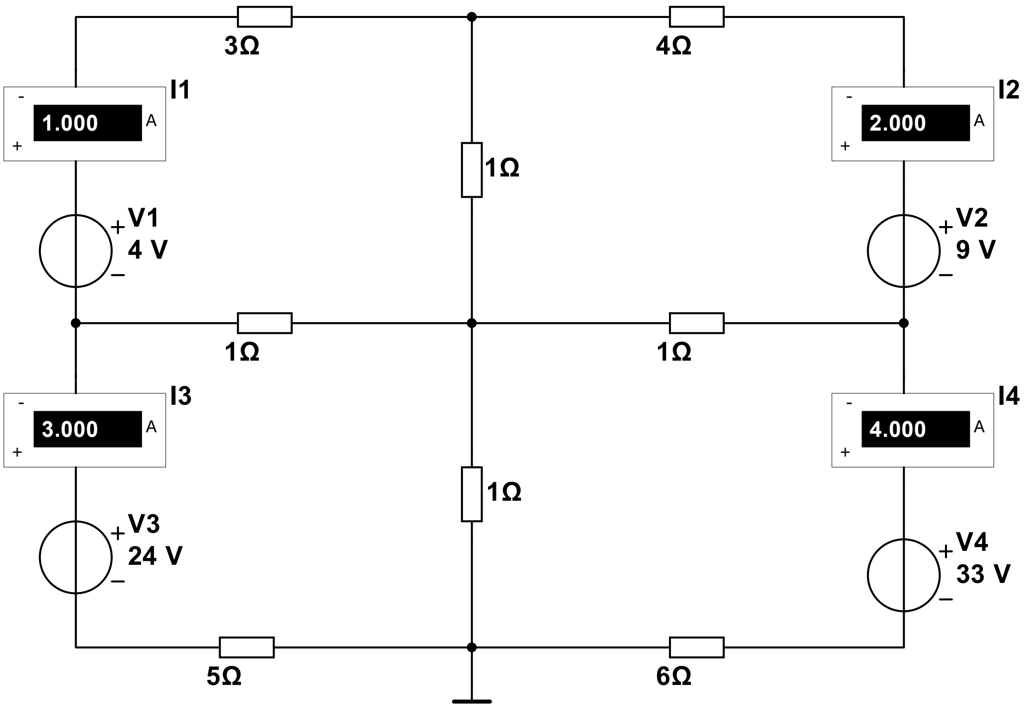

Fig. 6 The circuit simulation using Multisim for the four-dimensional instance